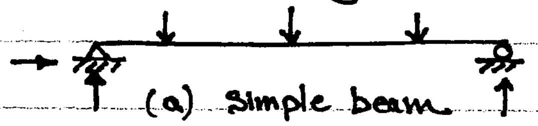
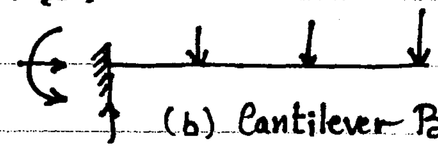
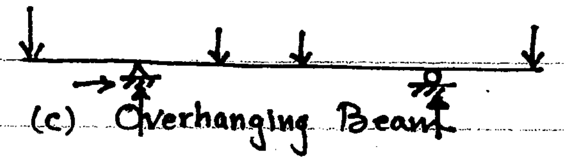
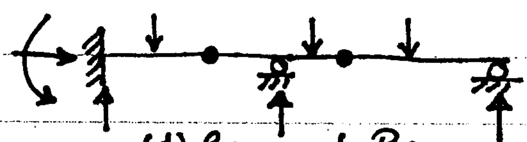
Statically Determinate Beams
– Shear & Moments –
(Review Statics).

Types of Statically Determinate Beams.


(a) simple beam

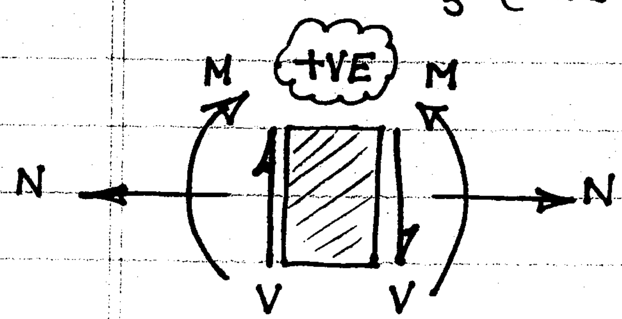

(b) Cantilever Beam

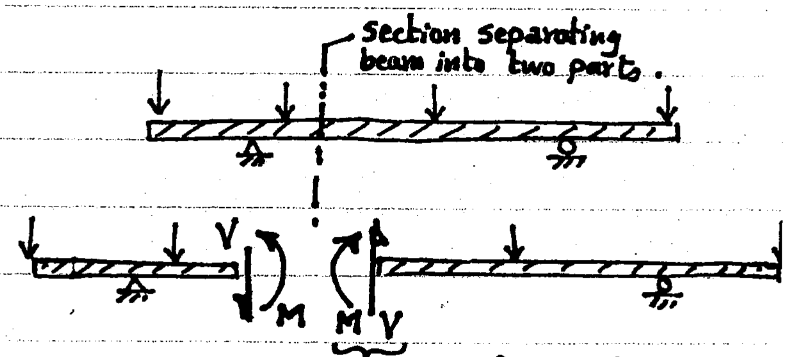

(c) Overhanging Beam


(d) Compound Beam
(Beam with internal hinges)

* Analysis ⟹ Get Internal Forces ⟹ Design sections according to design co
(N, V, M)

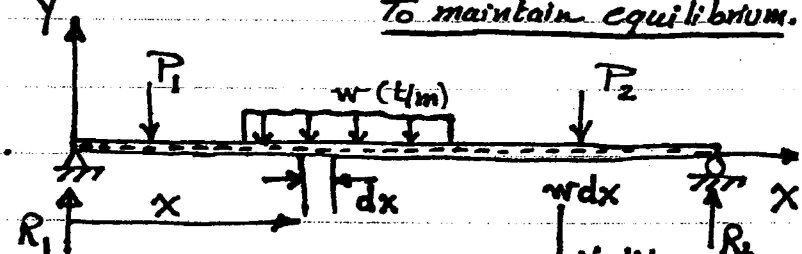
Shear & Bending Moments.





Effect of the left part on the right part. (Internal Forces).
To maintain equilibrium.

SIGN CONVENTION.



* For the free body diagram below.

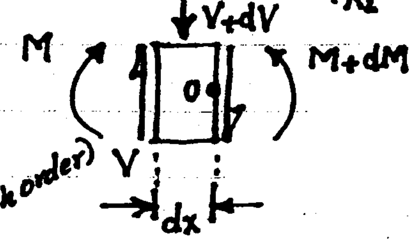
$\Sigma F_y = 0.$

$\Rightarrow V - w dx - (V + dV) = 0$

$$\Rightarrow \boxed{w = -\frac{dV}{dx}} \Rightarrow \int dV = -\int w dx.$$

$$\Rightarrow \Delta V = -\int w . dx.$$

$V_{max} \rightarrow$ when $w = 0$; $V_2 - V_1 = -w \int dx.$

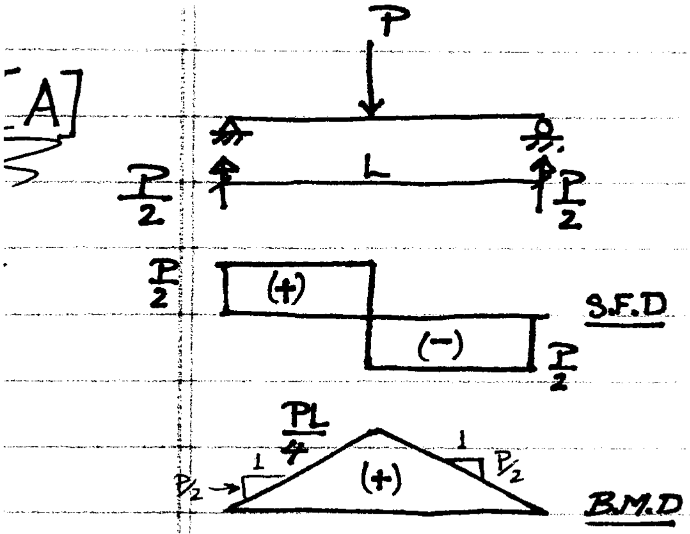


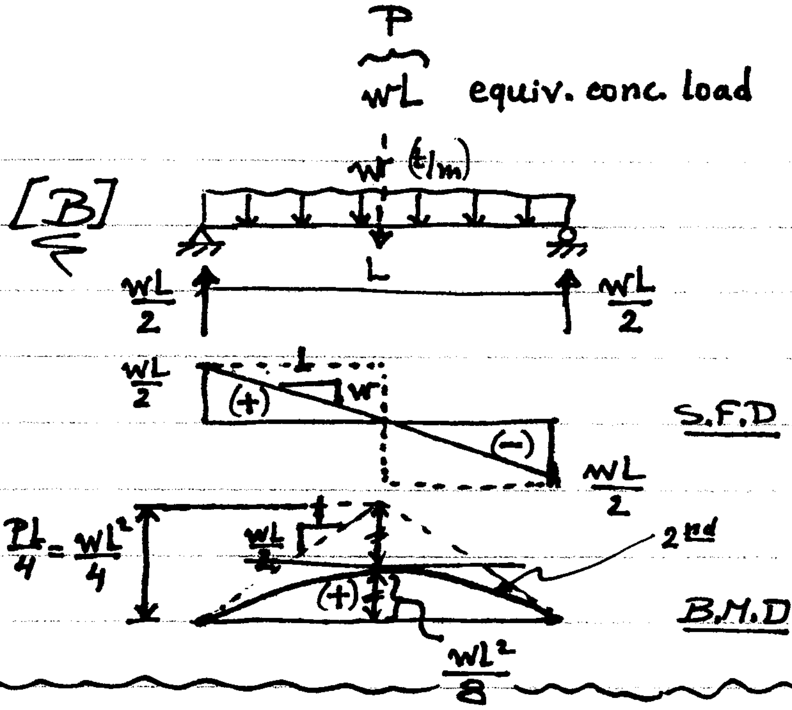
Free body diagram of an element of length dx.

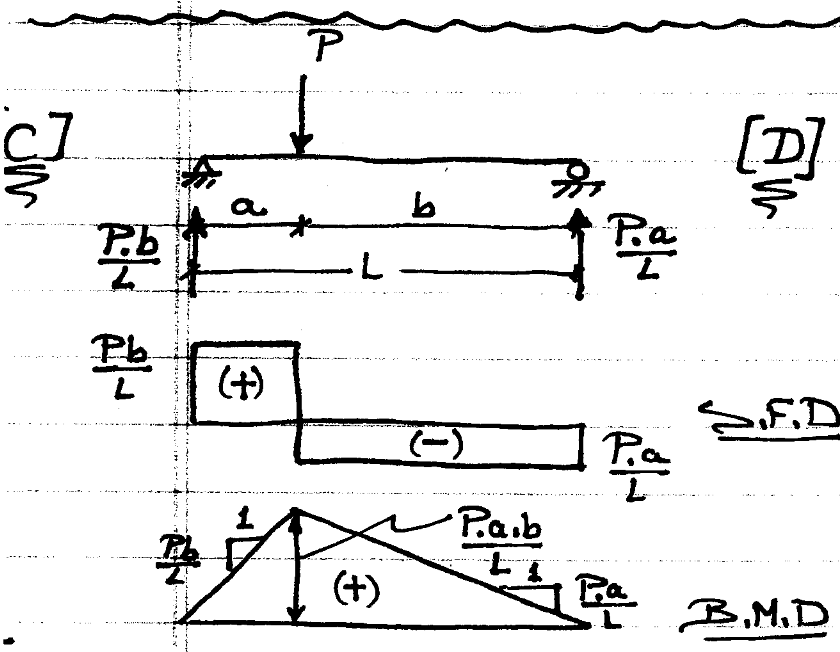
$$\Sigma M_0 = 0 \Rightarrow M - (M + dM) + V dx - w . dx . \underbrace{\frac{dx}{2}}_{\simeq 0 \text{ (high order)}} = 0$$

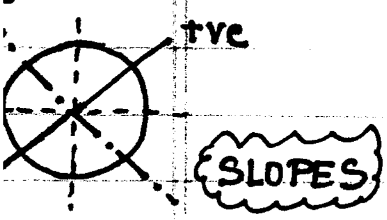
$$\Rightarrow \boxed{\frac{dM}{dx} = +V} \Rightarrow \int dM = \int V dx = -\iint w dx . dx .$$

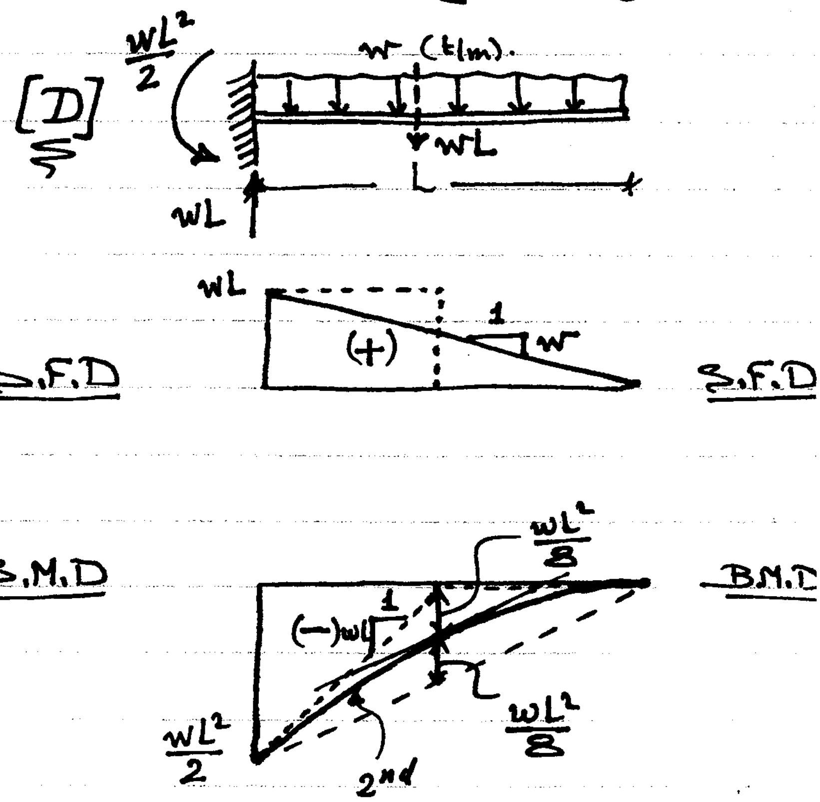
$M = max \longrightarrow$ when $V = 0$. $\quad M_2 - M_1 = \int V \times dx$

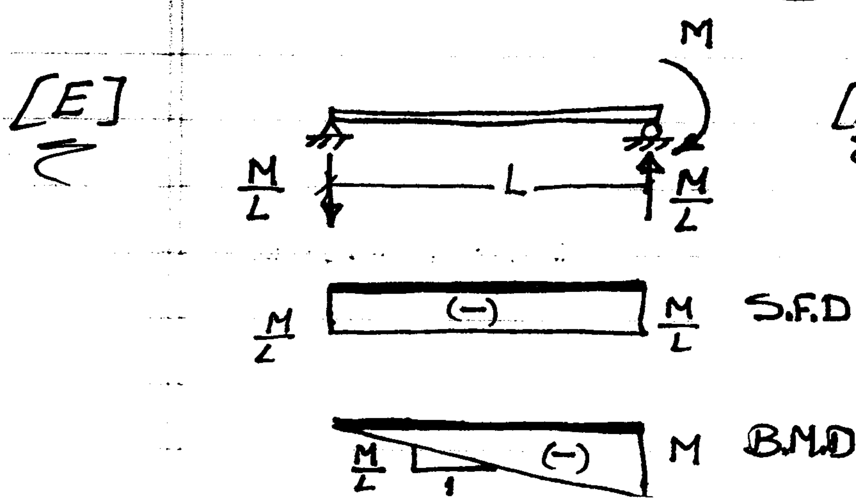

[A]
P
L
P/2
P/2
(+)
(−)
S.F.D
PL/4
1
P/2
(+)
B.M.D

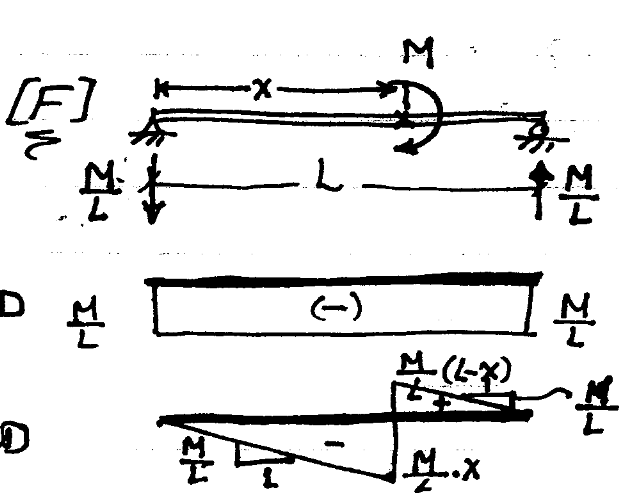

P
wL
equiv. conc. load
w (t/m)
[B]
L
wL/2
wL/2
1
w
(+)
(−)
S.F.D
PL/4 = wL²/4
wL²/8
2nd
(+)
B.M.D


P
[C]
a
b
L
P.b/L
P.a/L
Pb/L
(+)
(−)
S.F.D
P.a/L
1
P.a.b/L
Pb/L
(+)
P.a/L
B.M.D

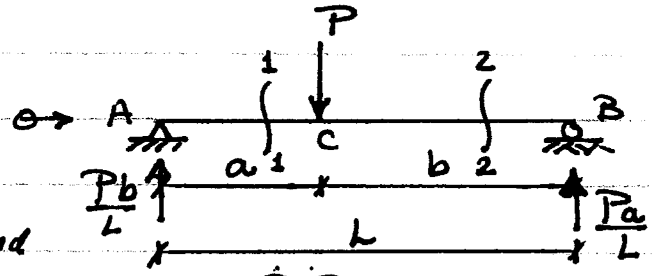

+ve
SLOPES


wL²/2
w (t/m)
[D]
wL
L
wL
wL
1
w
(+)
S.F.D
wL²/8
(−) wL
1
wL²/8
wL²/2
2nd
B.M.D


M
[E]
L
M/L
M/L
(−)
M/L
M/L
S.F.D
M/L
1
(−)
M
B.M.D

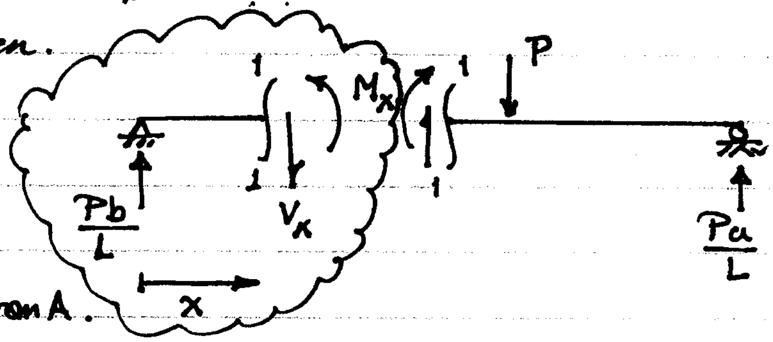

M
[F]
x
L
M/L
M/L
(−)
M/L
M/L
M/L (L−x)
+
M/L
−
M/L
1
M/L .x

Example



Obtain the eqns of shear "V" and moment "M" for the beam given.

- $\Sigma M_{/A} = 0 \Rightarrow R_B = \frac{P.a}{L}$
- $\Sigma y = 0 \Rightarrow R_A = \frac{P.b}{L}$.
- $\Sigma X = 0 \Rightarrow X_A = 0$



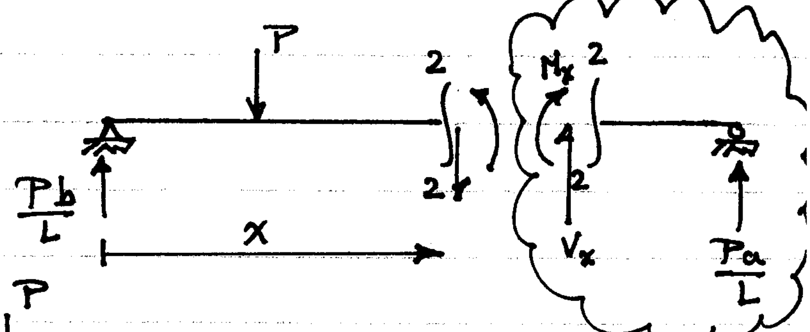
* Consider section 1-1, at x from A.

$$\left.\begin{array}{l} \Rightarrow V_x = \frac{Pb}{L} \;\ldots\; (\text{ct}). \\ \& \quad M_x = \frac{P.b}{L} * x. \;\ldots\; (\text{Str. Line}). \end{array}\right\} \Rightarrow \text{for } \underline{0 < x < a}.$$

* Now, for $\underline{a < x < L}$, consider section 2-2.

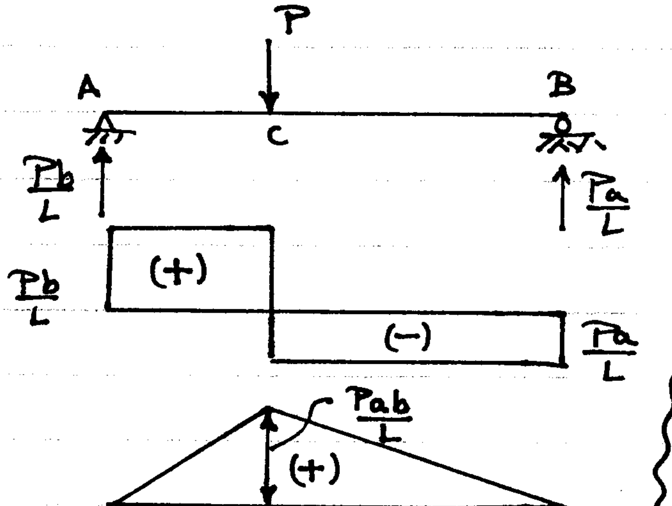
$$\Rightarrow V_x = -\frac{Pa}{L}.$$

$$\& \quad M_x = \frac{Pa}{L} * (L - x)$$



at $x = a$

$$\Rightarrow M_c = \frac{P.a.b}{L}.$$



S.F.D

$$\frac{dM}{dx} = V \Rightarrow M = \int V dx. = -\iint w dx.dx.$$

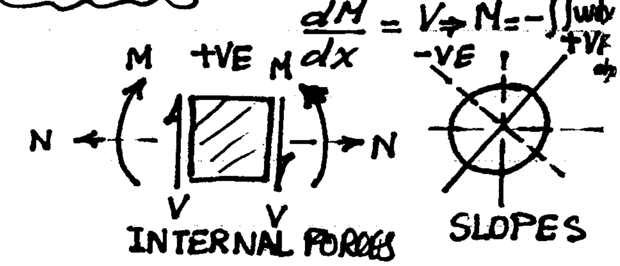
$$\text{OR} \quad \Delta M = M_2 - M_1 = \int_1^2 V dx$$

$$= \Delta \text{Area in S.F.D}\Big]_1^2$$

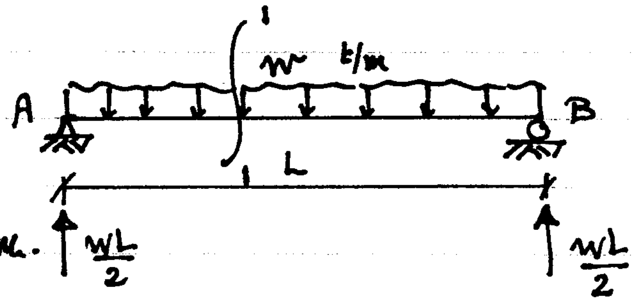
RECALL

$$\frac{dV}{dx} = -w \Rightarrow V = -\int w dx$$

$$\frac{dM}{dx} = V \Rightarrow M = -\iint w dx$$



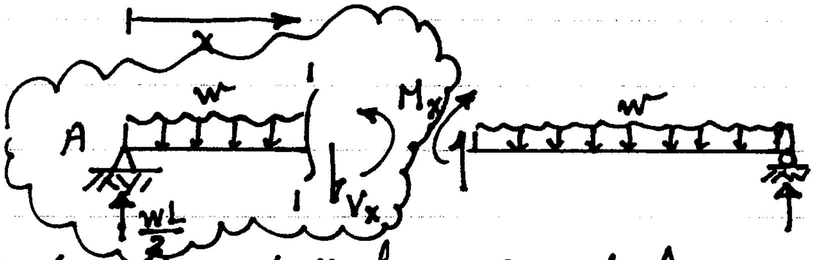
Example



Obtain the eqns of shear "V" & "M", moment, for the given beam.

Reactions.

- $\Sigma M_{/A}=0 \Rightarrow Y_B = \frac{wL}{2}$
- $\Sigma Y=0 \Rightarrow Y_A = \frac{wL}{2}$
- $\Sigma X=0 \Rightarrow X_A = 0.$



Consider a general section 1-1 at x from support A.

Equilibrium of left-hand side.

$$\Rightarrow \Sigma Y=0 \Rightarrow V_x = \frac{wL}{2} - w.x = w\left(\frac{L}{2}-x\right) \quad \text{--- Eqn of Str. Line}$$

$$\Rightarrow \Sigma M_{/A}=0 \Rightarrow M_x = +w.x.\frac{x}{2} + V_x(x)$$

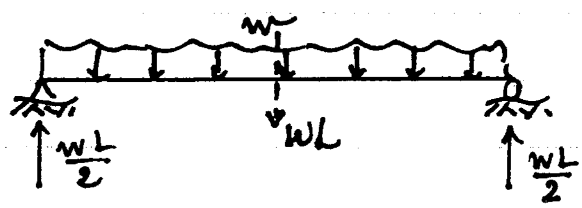
$$= w.\frac{x^2}{2} + w\left(\frac{L}{2}-x\right).x = w.\frac{x^2}{2} + \frac{wL}{2}x - w.x^2$$

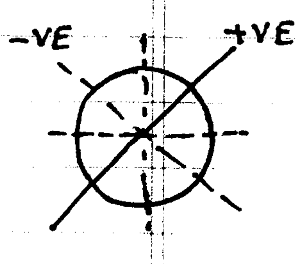
$$= \frac{wL\,x}{2} - \frac{wx^2}{2} = \frac{w}{2}(x)(L-x) \quad \text{.... Eqn of } 2^{nd} \text{ deg Parabola}$$

Verify $\frac{dM}{dx} = V$ & $\frac{dV}{dx} = -w$ ok.

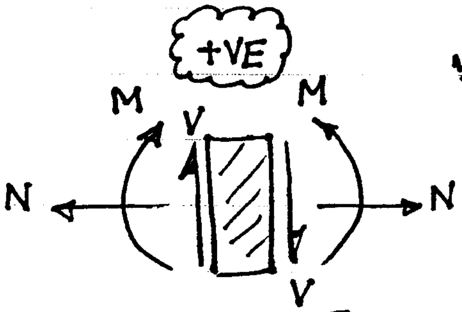
M_{max} is when $\frac{dM}{dx} = V = 0$

$\Rightarrow x = \frac{L}{2} \Rightarrow M_{max} = \frac{wL^2}{8}.$

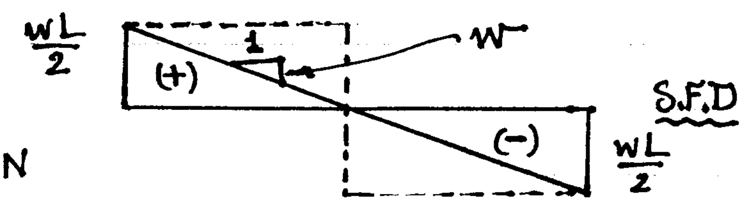


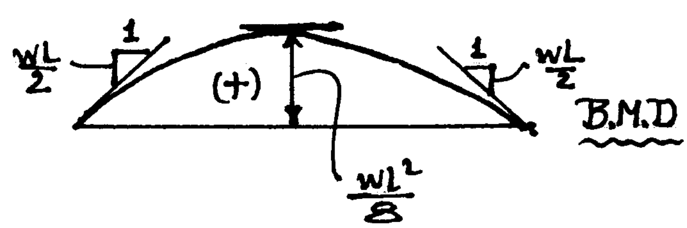


SLOPES

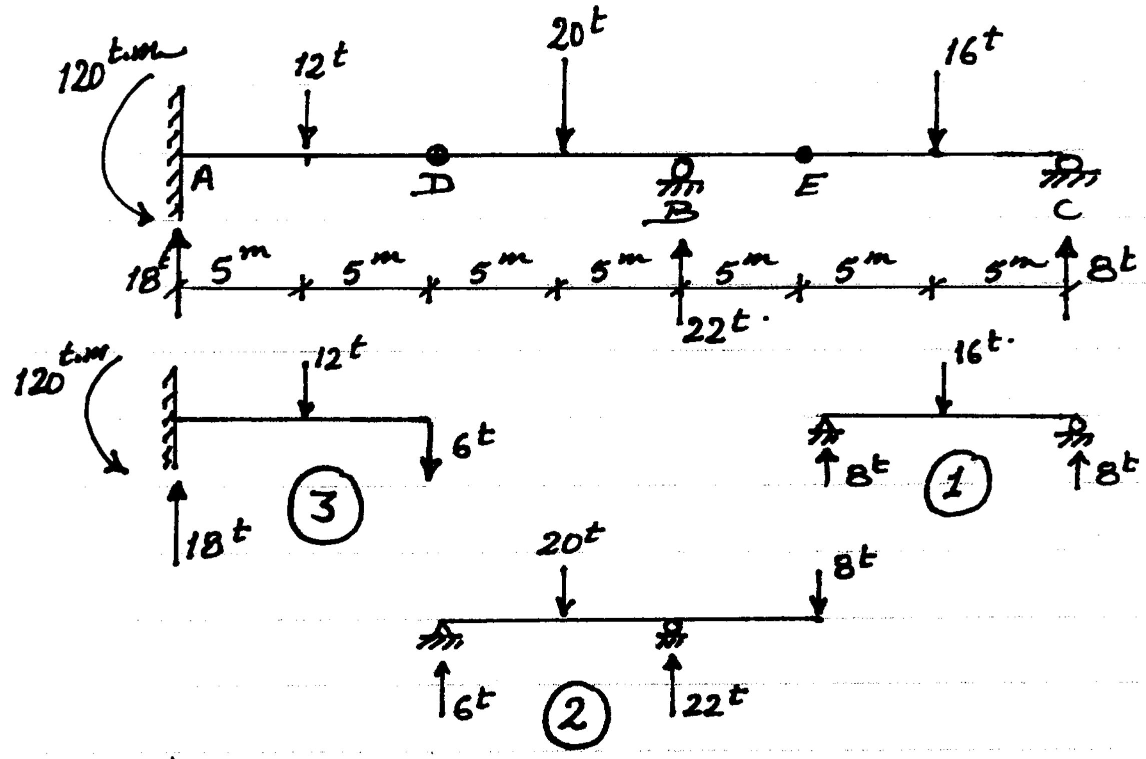


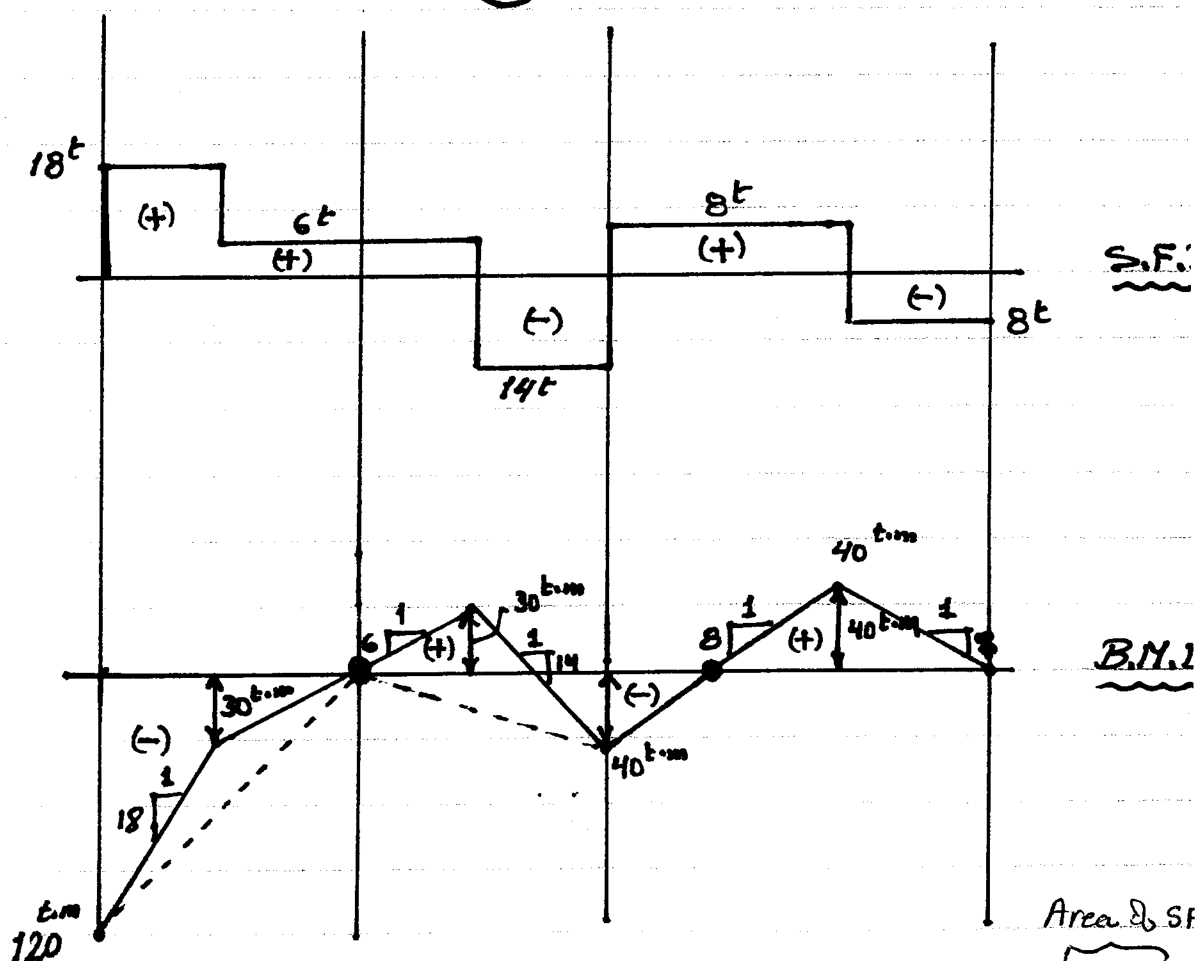
INTERNAL FORCES





Example





Area of SF

$$\Delta M = M_2 - M_1 = V \cdot \Delta x \Rightarrow M_2 = M_1 + \overbrace{V \cdot \Delta x}$$

$$\Rightarrow M_2 = -120 + 18(5) = -30 \, t.m.$$

⋮

etc.